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# Supersymmetric Domain Wall World from D=5 Simple Gauged Supergravity

Klaus Behrndt<sup>a1</sup> and Mirjam Cvetič<sup>b2</sup>

<sup>a</sup> *California Institute of Technology  
Pasadena, CA 91125*

<sup>b</sup> *Department of Physics and Astronomy  
University of Pennsylvania, Philadelphia, PA 19104-6396  
and  
Institute for Theoretical Physics  
University of California, Santa Barbara, CA 93106*

## Abstract

We provide explicit examples of supersymmetric domain walls of a five-dimensional simple, N=2 U(1) gauged supergravity theory constructed by Gunaydin, Townsend and Sierra. These conformally flat solutions interpolate between supersymmetric isolated anti-deSitter vacua, satisfy the Killing spinor (first order) differential equations, have the energy density related to the cosmological constants of the isolated supersymmetric vacua, and the four-dimensional world on the domain wall is a flat world with N=1 supersymmetry. An analysis of such solutions is given for the example of one (real, neutral) vector supermultiplet with the most general form of the prepotential. As the gauge coupling becomes very large (compared to five-dimensional Planck constant), these domain walls become infinitely thin, and a special case of a  $Z_2$  symmetric domain wall is a supersymmetric realization of the static domain wall solution considered by Randall and Sundrum.

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<sup>1</sup>e-mail: [behrndt@theory.caltech.edu](mailto:behrndt@theory.caltech.edu)

<sup>2</sup>e-mail: [cvetic@cvetic.hep.upenn.edu](mailto:cvetic@cvetic.hep.upenn.edu)

# 1 Introduction

The past few months have witnessed exciting progress in the study of domain walls in  $D=5$  gravity theories. Such configurations are interesting from two, on a surface or orthogonal perspectives: (i) in the context of AdS/CFT correspondence such conformally flat configurations provide new insights in the study of RGE flows [1, 2, 3, 4, 5, 6, 7, 8, 9] and (ii) in the context of phenomenological implications, such configurations provide a framework [10, 11, 12, 13, 14, 15, 16] to address the physics implications of large dimensions for the four-dimensional world on the domain wall.

Within the first approach a number of conformally flat solutions were constructed and in particular the ones interpolating between supersymmetric anti-deSitter (AdS) vacua of  $N=8$   $D=5$  gauged theory provide examples of static domain walls in  $D=5$  with implications for the renormalization group flow and spectra in strongly coupled four-dimensional super Yang-Mills theories. One such example [2, 5] involves two scalar fields and thus was solved numerically and another most recent example with one scalar field can be solved explicitly [17].

Within the second approach infinitely thin, static,  $Z_2$ -symmetric domain wall solutions were constructed [11, 12] for pure AdS gravity theory. (Generalizations that incorporate effects of additional compactified dimensions were given in [13, 15, 16].) These solutions have to satisfy a specific relation between the domain wall tension  $\sigma$  and the cosmological constant  $\Lambda$  of the AdS vacua, thus implying a fine-tuning.

The purpose of this letter is few-fold. We shall provide explicit examples of supersymmetric (BPS-saturated) domain walls in five-dimensions, in the simplest supergravity theory, i.e. the supergravity theory with least supersymmetry that allows for the explicit constructions of supersymmetric domain wall configurations. In order to demonstrate the existence of domain wall solutions in this framework, such a supergravity theory necessarily has to have a potential for (gauge neutral) scalar fields, and the only known such examples are gauged supergravity theories. We thus choose to work within a framework of a five-dimensional  $N=2$ ,  $U(1)$  gauged supergravity formulated by Gunaydin, Townsend and Sierra [18]<sup>3</sup>. For the sake of concreteness we shall analyse the case with one physical (gauge neutral) vector superfield which allows for an explicit analysis of the possible domain wall configurations. For this simple model, kink solutions have been discussed some time ago [19]; the domain walls presented in this paper provide a concrete and explicit realisation of supersymmetric kink solutions.

The important upshot of the analysis is that this framework does provide examples of static domain walls, i.e. conformally flat solutions interpolating between isolated supersymmetric vacua with (non-positive) cosmological constants. Those are BPS-saturated configurations that satisfy the Killing spinor (first order) differential equations and whose energy density  $\sigma$  is related in a specific manner to cosmological constants of

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<sup>3</sup>Such a theory may have its origin as a compactification of Type IIB superstring theory on a specific Einstein-Sasaki-5-manifolds or as a compactification of M-theory on Calabi-Yau with non-trivial fluxes turned on. But the details are unknown so far and remain to be worked out.

the isolated supersymmetric vacua on each side of the wall. Namely, the relationship between the domain wall tension and the cosmological constant is a consequence of the BPS nature of the solution, and not an artifact of fine-tuning<sup>4</sup>. These configurations have four unbroken supercharges, or in other words break  $\frac{1}{2}$  of N=2, D=5 supersymmetry, and thus the four-dimensional world on the domain wall has N=1 supersymmetry. A special example of infinitely thin BPS domain walls (corresponding to the case of very large gauge coupling) and with  $Z_2$  symmetry corresponds to a concrete supersymmetric realization of the static domain wall solution found by Randall and Sundrum [11].

## 2 D=5 N=2 U(1) Gauged Supergravity

Supergravity in D=5 is very restrictive with respect to allowed potentials. The only allowed potentials come from gauging of isometries and especially interesting are potentials that have no “run-away” behavior (scalars become asymptotically constant) with non-trivial isolated extrema. This type of potential allows for the existence of domain walls with extrema corresponding to the AdS vacua on each side of the wall. The minimal gauged supergravity (N=2 gauged supergravity with  $U(1)$  gauged  $R$ -symmetry), constructed in [18, 22], provides such a set-up. In this case one can consistently decouple the hyper-multiplets and the Lagrangian contains only the supergravity multiplet and the vector supermultiplets. (There are also domain wall solutions, that couple to non-trivial hypermultiplets [23], but they do not have asymptotic anti-de Sitter spaces.) In this case the bosonic Lagrangian reads:

$$S_5 = \int \left[ \frac{1}{2}R + g^2 V - \frac{1}{4}G_{IJ}F_{\mu\nu}^I F^{\mu\nu J} - \frac{1}{2}g_{AB}\partial_\mu\phi^A\partial^\mu\phi^B \right] + \frac{1}{48} \int C_{IJK}F^I \wedge F^J \wedge A^K \quad (1)$$

We chose the convention where the five-dimensional Newton’s constant is  $\kappa = 1$  and  $g$  is the gauge coupling. We work in the  $(-, +, +, +, +)$  convention. The physical scalars  $\phi^A$ , which are real and neutral, correspond to the scalar components of the vector super-multiplets and define coordinates of the manifold defined by [18]

$$F = \frac{1}{6}C_{IJK}X^I X^J X^K = 1, \quad (2)$$

with  $C_{IJK}$  real, and the  $X^I$  are the auxiliary real scalar fields. The metric(s) of the scalar manifold  $g_{AB}$  (for physical scalars  $\phi^A$ ) and  $G_{IJ}$  (for auxiliary scalars  $X^I$ ) are defined by

$$G_{IJ} = -\frac{1}{2}(\partial_I\partial_J \log F)_{F=1}, \quad g_{AB} = (\partial_A X^I \partial_B X^J G_{IJ})_{F=1} \quad (3)$$

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<sup>4</sup>These properties are very much parallel to those of supersymmetric domain walls of four-dimensional N=1 supergravity theory found in [20] and reviewed in [21].

where  $\partial_A \equiv \frac{\partial}{\partial \phi^A}$ .  $\partial_I \equiv \frac{\partial}{\partial X^I}$ . The auxiliary scalars  $X^I$  are accompanied by gauge field strengths  $F_{\mu\nu}^I$  entering the Lagrangian (1).

The gauging of a  $U(1)$  subgroup of the  $R$ -symmetry introduces a potential for the scalars<sup>5</sup>

$$\begin{aligned} V &= 6 h_I h_J \left( X^I X^J - \frac{3}{4} g^{AB} \partial_A X^I \partial_B X^J \right) \\ &= 6 \left( W^2 - \frac{3}{4} g^{AB} \partial_A W \partial_B W \right), \end{aligned} \quad (4)$$

where  $h_I$  are real constants, specifying the Fayet-Iliopoulos(FI) terms, and the superpotential  $W$  is defined as

$$W = h_I X^I. \quad (5)$$

Notice,  $W$  is subject to the constraint (2) which makes it non-linear in the physical scalars  $\phi^A$ .

## Supersymmetry Transformations and BPS-Saturated Domain Walls

We are searching for supersymmetric (BPS) domain wall solutions: those are solutions that preserve part of the supersymmetry, and thus satisfy the Killing spinor equations, which are first order differential equations that ensure that the supersymmetry transformations in this domain wall background are preserved.

We chose these domain wall solutions to be neutral, and thus they are supported only by (gauge neutral) scalars with the gauge fields turned off. Thus, the supersymmetry transformations for these backgrounds read: [18]

$$\begin{aligned} \delta \lambda_A &= \left( -\frac{i}{2} g_{AB} \Gamma^\mu \partial_\mu \Phi^B + i \frac{3}{2} g \partial_A W \right) \epsilon, \\ \delta \psi_\mu &= \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \Gamma_{ab} + \frac{1}{2} g \Gamma_\mu W \right) \epsilon. \end{aligned} \quad (6)$$

The vacuum is given by the asymptotic space, where the scalars are constant and thus supersymmetry requires  $\partial_A W = 0$ . The form of the potential  $V$  (4) implies that supersymmetric vacua are always extrema of the potential.

The domain wall Ansatz for the metric is of the form:

$$ds^2 = A(z) \left[ -dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + dz^2, \quad (7)$$

and the scalars have the form  $\phi^A = \phi^A(z)$ , where  $z = \{-\infty, +\infty\}$  is a direction transverse to the wall.

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<sup>5</sup>Note the parallels with the potential in D=4 N=1 supergravity where:  $V = e^K \left( g^{A\bar{B}} D_A W \overline{D_B W} - 3|W|^2 \right)$  where  $W$  and  $K$  are the superpotential and Kähler potential for the chiral superfields.

Then the Killing spinor equations  $\delta\psi_\mu = 0$  and  $\delta\lambda_A = 0$  are solved by <sup>6</sup>:

$$\partial_z \log A = -2gW , \quad (8)$$

and

$$\partial_z \phi^A = -3gg^{AB}\partial_B W , \quad (9)$$

where the four component spinor satisfies the constraint:  $\Gamma_z \epsilon = -\epsilon$ . (Killing spinor equations for domain walls of D=5, N=8 supergravity can be cast in a similar form [5].) Note that as long as the domain of physical fields contain two isolated supersymmetric vacua, this set of solutions specify the BPS domain wall. The physical domain of such solutions requires that the scalar metric  $g_{AB}$  remains positive definite.

The domain wall tension can be determined by applying Nester's procedure which relates the wall tension  $\sigma$  to the central charge of the supersymmetry algebra; the central charge is determined by the values of the superpotential at each asymptotically supersymmetric vacuum. (For D=4 N=1 domain wall solutions, see Appendix A of [20].) More concretely, one considers the integral over the spatial boundary

$$\int_{\partial\Sigma} N^{\mu\nu} d\Sigma_{\mu\nu} = \int_{\Sigma} \nabla_\mu N^{\mu\nu} d\Sigma_\nu = \int_{\Sigma} \nabla_\mu N^{\mu 0} d\Sigma_0 . \quad (10)$$

Here we used the Stokes theorem.  $\Sigma_{\mu\nu}$  is a space-like hypersurface and thus  $d\Sigma_0 \sim dzd\vec{x}$ . The Nester tensor reads  $N^{\mu\nu} = \bar{\epsilon}\Gamma^{\mu\nu\lambda}\delta\psi_\lambda$  where  $\delta\psi_\lambda$  is the gravitino variation. In order to determine the energy density, we can factor out the integral over the domain wall coordinates ( $d\vec{x}$ ) and the integration over the transverse direction ( $z$ ) yields in (10) the contributions far away from the wall. Inserting the gravitino variation in (10), one obtains two contributions, the first one represents the domain wall tension  $\sigma$  (energy density) and the second one corresponds to the central charge  $\mathcal{C}$ . The latter one is a topological term that corresponds to the difference of the boundary values of the superpotential. For the supersymmetric configuration the gravitino variation is zero, and thus (10) is zero which implies:

$$\sigma = \mathcal{C} \equiv \frac{1}{12}(\bar{\epsilon}\sqrt{A}\Gamma^0\epsilon)gW\Big|_{+\infty}^{-\infty} , \quad (11)$$

where we used the projector  $\Gamma_z \epsilon = -\epsilon$ . (In the Killing spinor equations and (11) we chose  $W(\phi^A|_{z=-\infty}) \geq W(\phi^A|_{z=+\infty})$ . The reversal of this inequality implies the change of signs in the Killing spinor equations, (11) and  $\Gamma_z \epsilon = \epsilon$ .)

Normalizing the Killing spinor as  $(\bar{\epsilon}\Gamma^0\sqrt{A}\epsilon) = 1$ , yields the result is:

$$\sigma_{BPS} = \frac{g}{12}|W_{+\infty} - W_{-\infty}| = \frac{1}{12\sqrt{6}}|\sqrt{-\Lambda_{+\infty}} \pm \sqrt{-\Lambda_{-\infty}}| , \quad (12)$$

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<sup>6</sup>In D=4 N=1 supergravity, the Killing spinor equations are similar [20]:  $\partial_z \log A \sim e^{\frac{K}{2}}W$  ,  $\partial_z \phi^A \sim e^{\frac{K}{2}}g^{A\bar{B}}\overline{D_B W}$ .

where  $W_{\pm\infty} \equiv W(\phi^A|_{z=\pm\infty})$ . In the second part of (12) we have used the relationship between the cosmological constant  $\Lambda$  and the value of the superpotential  $W$  for supersymmetric vacua. Note that  $\pm$  is determined by  $\text{sign}[W_{+\infty}W_{-\infty}]$ . Thus, the domain wall tension is specified by the values of the cosmological constants of the asymptotic AdS vacua. According to (12) there are three types of BPS-saturated domain walls (very much parallel to the analysis of the types of BPS-saturated domain walls in D=4 [25], their global space-time structure [26] and their relationship to non-supersymmetric configurations [27]):

- Type I domain walls interpolate supersymmetric Minkowski space-time ( $\Lambda_{-\infty} = 0$ ) and the AdS space time ( $\Lambda_{+\infty} \equiv \Lambda \neq 0$ ). In this case  $\sigma_{BPS} = \frac{1}{12\sqrt{6}}\sqrt{-\Lambda}$ . These walls saturate an analog of the Coleman-DeLuccia bound [28] in five dimensions. On the asymptotic AdS side the metric coefficient takes the form  $A(z) = e^{-\sqrt{-\frac{2}{3}\Lambda_{\pm\infty}}z}$ ; on the AdS side of the wall  $z \rightarrow \infty$  limit corresponds to the AdS horizon. The geodesic extension of these space-times could either be pure AdS or new regions that involve an infinite tiling with the “mirror” domain walls.
- Type II domain walls interpolate between supersymmetric AdS vacua where  $\text{sign}[W_{-\infty}] = -\text{sign}[W_{+\infty}]$  and thus  $\sigma_{BPS} = \frac{1}{12\sqrt{6}}(\sqrt{-\Lambda_{+\infty}} + \sqrt{-\Lambda_{-\infty}})$ , and the asymptotic metric behaves as  $A(z) = e^{-\sqrt{-\frac{2}{3}\Lambda_{\pm\infty}}|z|}$ ; on each side of the walls  $z \rightarrow \pm\infty$  limits correspond to the AdS horizons and thus the geodesic extensions could be either pure AdS or new regions that comprise of an infinite tiling with the mirror domain walls. (These domain walls can be viewed as “stable”; non-supersymmetric generalizations are expanding bubbles on either side of the AdS vacua [24].) A special case of a  $Z_2$  symmetric solution ( $W_{+\infty} = -W_{-\infty}$ ) satisfies the constraint:  $\sigma_{BPS} = \frac{1}{6\sqrt{6}}\sqrt{\Lambda}$  which is a relationship found in [11].
- Type III domain walls are those between two supersymmetric AdS vacua where  $\text{sign}[W_{-\infty}] = +\text{sign}[W_{+\infty}]$  and thus  $\sigma_{BPS} = \frac{1}{12\sqrt{6}}|\sqrt{-\Lambda_{+\infty}} - \sqrt{-\Lambda_{-\infty}}|$ . The metric coefficient grows exponentially on one side of the wall:  $A(z) = e^{+\min[\sqrt{-\frac{2}{3}\Lambda_{\pm\infty}}]|z|}$ , and thus on this side,  $|z| \rightarrow \infty$  limit corresponds to the boundary of the AdS space! (Those are the “unstable” domain wall solutions whose non-supersymmetric generalizations corresponds to false vacuum decay bubbles, only [24].)

### 3 BPS Domain Walls with One Vector Supermultiplet

For the sake of being explicit we will address the case of a single vector multiplet. Defining the physical scalar as  $\phi = X^1/X^0$  the constraint (2) takes the form:

$$F = (X^0)^3 (A + B\phi + C\phi^2 + D\phi^3) = 1, \quad (13)$$

and the superpotential (5) becomes:

$$W = X^0(h_0 + h_1\phi) . \quad (14)$$

where  $X^0$  is the auxiliary field eliminated by eq. (13). The metric  $g_{\phi\phi}$ , and the derivative of the potential  $\partial_\phi W$  take the form:

$$g_{\phi\phi} = \frac{1}{3} \frac{(C^2 - 3BD)\phi^2 + (BC - 9AD)\phi + (B^2 - 3AC)}{(A + B\phi + C\phi^2 + D\phi^3)^2} , \quad (15)$$

$$\partial_\phi W = \frac{(\frac{1}{3}h_1C - h_0D)\phi^2 + \frac{2}{3}(h_1B - h_0C)\phi + h_1A - \frac{1}{3}h_0B}{(A + B\phi + C\phi^2 + D\phi^3)^{4/3}} , \quad (16)$$

and the potential reads:

$$V = 6 \left[ W^2 - \frac{3}{4g_{\phi\phi}} (\partial_\phi W)^2 \right] . \quad (17)$$

One can make the following general observations about the nature of supersymmetric vacua. (i) The superpotential (14) allows for at most two extrema, where  $\partial_\phi W = 0$ . (ii) Expanding  $W$  around a given extremum yields:  $\partial_\phi^2 W|_{extr} = \frac{2}{3} g_{\phi\phi} W|_{extr}$  (see e.g., [6]). This relationship implies that for physical solutions with  $g_{\phi\phi} > 0$ , the two extrema of  $W$  cannot be connected unless there is at least one pole between them. Thus, the supersymmetric domain wall solution necessarily involves a “jump” over a region where the superpotential (as well as the scalar metric and the potential) have a pole. As we shall see shortly such poles are sufficiently mild that the domain wall solutions, analyzed in the previous section exist and the domain wall tension is finite. (Note, these lines of arguments hold only for the one-scalar case. If  $W$  depends on more than one scalar, it may allow for two minima, which can be smoothly connected.)

In order to discuss the solution in more detail we choose, without loss of generality, the following parameterization:

$$g = 1, \quad A = 0, \quad B = D = h_0 = 1, \quad C = \sqrt{3}\chi, \quad h_1 = \sqrt{3}\xi . \quad (18)$$

(One can show that  $g = D = h_0 = 1$  corresponds to an overall rescaling of the potential,  $A = 0$  can be obtained by shifts  $\phi \rightarrow \phi - \phi_0$  and  $h_0 \rightarrow h_0 + h_1\phi_0$ , and  $B = 1$  corresponds to a rescaling of  $\phi$ .) In this case the metric, superpotential and its derivate can be written in the following form:

$$g_{\phi\phi} = \frac{3(\chi^2 - 1)\phi^2 + \sqrt{3}\chi\phi + 1}{3\phi^2(1 + \sqrt{3}\chi\phi + \phi^2)^2} , \quad (19)$$

$$W = \frac{1 + \sqrt{3}\xi\phi}{[\phi(1 + \sqrt{3}\chi\phi + \phi^2)]^{\frac{1}{3}}} , \quad (20)$$

$$\partial_\phi W = \frac{3(\chi\xi - 1)\phi^2 - 2\sqrt{3}(\chi - \xi)\phi - 1}{3[\phi(1 + \sqrt{3}\chi\phi + \phi^2)]^{\frac{4}{3}}} . \quad (21)$$

The corresponding Killing spinor equations for the metric coefficient  $A(z)$  is given in (8) and that for the scalar field (9) takes the form:

$$g_{\phi\phi} \partial_z \phi = -3 \partial_\phi W . \quad (22)$$

## Features of the Solutions and an Example

The first useful observation is that in the region where the metric  $g_{\phi\phi}$  has real poles,  $g_{\phi\phi}$  has no real zeroes. Namely, the poles and zeros are at the following values of  $\phi$ :

$$\text{poles of } g_{\phi\phi} : \left\{ \frac{1}{2}(-\sqrt{3}\chi \pm \sqrt{3\chi^2 - 4}), 0 \right\}, \quad (23)$$

$$\text{zeroes for } g_{\phi\phi} : \frac{-\chi \pm \sqrt{-3\chi^2 + 4}}{2\sqrt{3}(\chi^2 - 1)}. \quad (24)$$

Thus for  $\chi^2 > 4/3$  there are *no* zeroes of the metric, but there are poles for the values of  $\phi$  specified by (23).

Supersymmetric vacua are determined by zeroes of  $\partial_\phi W$  (21). As discussed at the beginning of this section, there are at most two, and there  $\phi$  takes the value:

$$\text{zeroes for } \partial_\phi W : \frac{(\chi - \xi) \pm \sqrt{\chi^2 - \xi\chi + \xi^2 - 1}}{\sqrt{3}(\xi\chi - 1)}. \quad (25)$$

Note also that the poles of  $W$ ,  $\partial_\phi W$  and  $g_{\phi\phi}$  are identical.

For the parameter range  $\chi^2 < \frac{4}{3}$ ,  $W$  has no poles and thus, due to the relationship  $\partial_\phi^2 W|_{extr} = \frac{2}{3} g_{\phi\phi} W|_{extr}$ , one extremum has to be a maximum and the other one a minimum. Therefore, the scalar metric  $g_{\phi\phi}$  is negative for one value of (25) and corresponds to a non-physical vacuum ( $\phi$  is a tachyon there).

Thus the only physical region for the domain wall solutions corresponds to  $\chi^2 > \frac{4}{3}$ . Now the scalar metric  $g_{\phi\phi}$  is always positive definite and  $W$  has two real extrema which are necessarily separated by at least one pole. Thus the domain wall interpolating between such supersymmetric extrema corresponds to a kink solution that goes over the region where the potential blows-up mildly. Near the pole the Killing spinor equations (8) and (9) can be solved approximately: instead of a typical kink behavior  $\phi - \phi_{pole} \sim (z - z_{pole})$  (in the case of a finite potential) now the kink “slows-down” and behaves near the pole as  $\phi - \phi_{pole} \sim (z - z_{pole})^3$  and the metric coefficient behaves as <sup>7</sup>  $A(z) \sim (z - z_{pole})^{2c}$  where  $c$  depends on the pole-value of  $\phi$ . It is negative for  $\xi > \frac{1}{3(\chi-1)}$  and positive for  $\xi < \frac{1}{3(\chi-1)}$ .

Typically one encounters Type II domain wall solutions, i.e.  $\text{sign}[W_{+\infty}] = -\text{sign}[W_{-\infty}]$ , and at least one pole in-between. (Further details will be given elsewhere [24].) For the sake of concreteness we exhibit a numerical solution for  $\chi = 1.4$ ,  $\xi = -0.6$  with the two supersymmetric minima (25) at  $\{-1.0887, -0.1664\}$  sandwiched between the pole in the middle (poles (23) are at  $\{-1.8980, -0.5269, 0\}$ ). This solution is close to a  $Z_2$  symmetric solution;  $W|_{-\infty} = 2.6944$  and  $W|_{+\infty} = -2.4953$ . Notice the “slow-down” of the kink solution  $\phi(z)$  and a power-law “spike” of the metric  $A(z)$  in the middle of the wall.

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<sup>7</sup>Note that the energy density contributing to the domain wall tension is finite and thus the Nester’s procedure for the derivation of  $\sigma_{BPS}$  (which employs Stoke’s theorem) remains applicable.



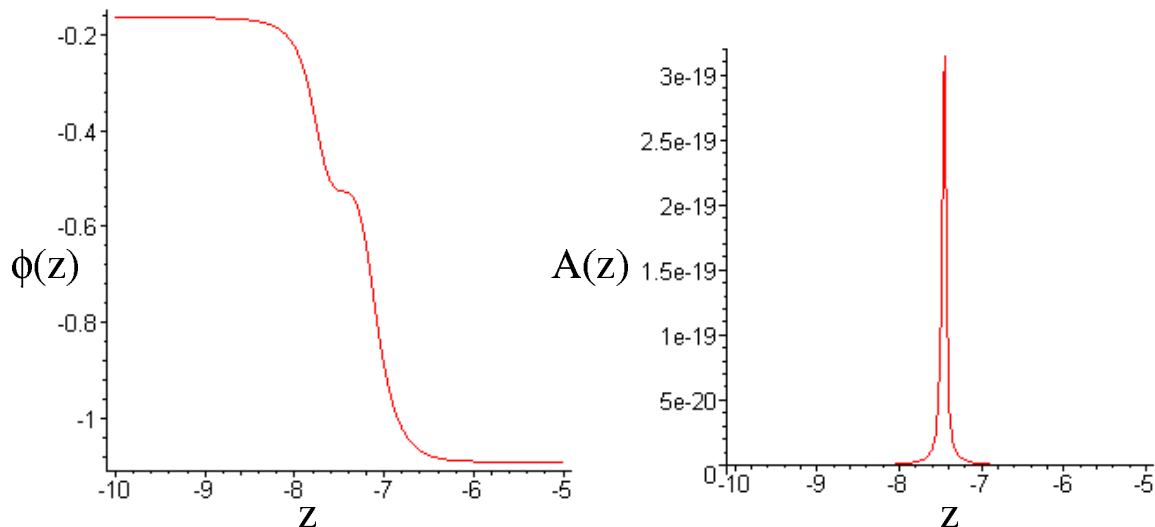


Figure 1: The domain wall solution for the scalar field  $\phi(z)$  and the metric coefficient  $A(z)$  is depicted for the choice of parameters  $\chi = 1.4$  and  $\xi = -0.6$ . Note the “slow-down” of the kink and a “spike” of the metric coefficient  $A(z)$  in the region in the middle of the wall.

## 4 Conclusions

The specific realization of supersymmetric domain walls in the simplest five-dimensional supergravity demonstrates a number of interesting features. The superpotential  $W$  as a function of a single scalar can have at most two extrema, but there is no smooth flow possible while demanding that the scalar metric remains positive ( $g_{\phi\phi} > 0$ ). Two AdS vacua with positive scalar metric have to be separated by a pole in the superpotential and the corresponding domain wall represents a supergravity kink solution that interpolates between the two branches. Despite this singularity, a stable kink solution exists (with the scalar field “slowing-down” mildly in the region crossing the pole) and the energy density, as calculated by Nester’s procedure, is finite and given by the sum or difference (depending on the relative signs of  $W$ ) of the asymptotic cosmological constants.

As for particle phenomenology, we have clarified an important connection between the domain wall tension and the cosmological constants of isolated D=5 string vacua. As a by product we see that the domain wall world is flat and is supersymmetric, i.e. along with the massless graviton there is an accompanying gravitino. The hypermultiplets of D=5 gauged supergravity could potentially play a role of matter on the domain wall, a subject of further investigations.

We conclude with another note, that the break-down of supersymmetry (by either of the vacua) would ensure that the non-extreme walls would become expanding bubbles

(see the analysis given in for non-supersymmetric domain walls in  $D=4$  in [27] and for a somewhat related analysis in  $D=5$  in [29].)

Could it be that the four-dimensional world is indeed a domain wall world of gauged supergravity theory?

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